**Bottoms Up**

**Please be careful with children handling the dice. Dice should never go in their mouth as they can be a choking hazard!**

**Dice Legend**

| Black Dice | Addition |
| --- | --- |
| Red Dice | Subtraction |
| Green Dice | Multiplication |
| Purple Dice | Square & square root |
| White Die | Operators, + - x |
| White Numbers  “Basic” | On 8-sided dice will have the numbers 2-9 |
| Yellow Numbers  “Advanced” | On 8-sided dice will have the numbers 6-13 |

**Type of Game:** BottomsUp is actually not a game, but rather a specific way to “use” the dice. In BottomsUp, players do whatever activity they would normally do with those dice (add the black ones, multiply the green, etc.) except that they do that process with the number on the bottom of the die rather than the one that they can see on the top.

**Who Should Play:** Players will not be able to “play” BottomsUp successfully until they can subtract numbers from 11,19 and 15. The way players learn what number is on the bottom of the die is simply by subtracting the top number from 11 (when using white numbered dice) and by subtracting the top number from 19 when using the yellow numbered dice. Players must subtract the top number from 15 when using either the perfect square or square root purple dice.There are many more practical uses of BottomsUp with the black, red and green dice than with the purple dice, but the same technique is used to find the “bottom” number no matter which dice you are using.

**Materials Needed:** You can learn BottomsUp with just one die. Simply roll it and then subtract the top number from 11, 15 or 19 to find the number on the bottom. No answer chart is needed. If the player is using the yellow numbered dice and rolls a 13 and does not remember what 19 minus 13 is, all they have to do is look at the bottom of the die.

**Skills Reinforced:** Basic BottomsUp is great practice for subtraction and then a tremendous confidence builder when players can easily figure out the number on the bottom of the dice and then do whatever process they are practicing, based on the color of the dice. Perhaps the greatest value of BottomsUp is actually not realized until later, when the players are learning to do algebra. BottomsUp provides great practice using algebraic formulas to solve problems, and kids will see the value of algebra when they realize that there is a formula that allows them to work with even multi-step BottomsUp problems without ever having to figure out what number is on the bottom of any die. Pretty cool, and players can come up with the exact formula to do any problem as soon as they learn the process/

* **How to play basic BottomsUp;** For addition, roll two basic black dice and figure out the number on the bottom and then add them. For example, if the player rolls a 6 and 7, the player figures out that the number on the bottom of the 6 is 5, since 11-6 =5 and then figures out that the number on the bottom of the 7 is 4 since 11 -7 = 4, and then the player adds the 5 and the 4 to get 9. Green dice are played exactly the same way, except that the numbers on the bottom are then multiplied together.
* When playing with yellow numbered dice, the procedure is exactly the same except that the top number is subtracted from 19 to get the bottom number, so if the player rolls a 12 and an 8 they know that the number on the bottom of the 12 is a 7 since 19-12 = 7 and the number on the bottom of the 8 is an 11 since 19-8 = 11, and then the player adds the 7 plus 11 to get 19.
* Of course, you can use two or three colors of dice and figure out the number on the bottom of each and then do whatever process is appropriate first, based on the rules for “order of operations.” (PEMDAS)
* Players can play any of the countless MathPro games (ClimbUp, ColorUp, RummyUp, any of them) in the BottomsUp format, although I am not sure why anyone other than a numbers maniac would choose to.
* **BottomsUp for using algebraic formulas:** There are way too many possible formulas to use to solve problems in BottomsUp to list them all here, but when using the black red and green dice they will all consist of (11-a) for the white numbered dice or (19-a) for the yellow numbered dice. a, b, c and so on will represent the top number in these formulas.
* So, let’s use some basic algebra to solve some BottomsUp addition problems without ever having to figure out what number is on the bottom of any of the dice. If we are playing with two basic black dice our formula would be: (11- a) + (11 - b) = the sum of the two numbers on the bottom. When we do the algebra and simplify it we get that 22 - a - b = the sum of the two numbers on the bottom. 22 -a -b can be rewritten as 22 - (a + b) = the sum of the numbers on the bottom, so, presto, we can figure out the sum of the numbers on the bottom simply by adding the two numbers on the top and subtracting them from 22. Learn the algebraic formula, and BottomsUp is easy. Yahoo for algebra! When playing BottomsUp with yellow numbered black dice the formula simply changes to (19 -a) + (19 - b) = the sum of the two numbers on the bottom, which simplifies to 38 - (a + b) = the sum of the two numbers on the bottom. And on we go. For two basic black dice and one advanced black die the formula is (11 - a) + (11 - b) + (19 - c) = the sum of the three numbers on the bottom, which simplifies easily to 41 - (a + b + c) = the sum of the three number on the bottom. Even if you can’t add those three numbers and subtract them from 41 in your head, if you know the formula you can do it a whole lot quicker on scratch paper (or with our dry erase) than you can by figuring out what is the bottom number on all of those dice and then adding them. The more dice you use, the more helpful the formula. Yahoo for algebra! Here are two more even more complex BottomsUp problems, and how to solve them using algebra.
* Let’s say you are going to add three black dice (two basic and one advanced) and from that total subtract two red dice (one basic and one advanced). And remember, all of this is being done in BottomsUp mode meaning we are going to add three numbers we cannot see and from that sum subtract two numbers that we cannot see. Of course, you can figure out what number is on the bottom of all five of those dice and add the three black numbers together and subtract the two red ones, or you can learn how to use an algebraic formula which at first will seem complicated, but in the end will be very easy to solve. And just for review, remember that the variables a, b and c will represent the three black numbers that we can see, and the variables d and e will represent the two red numbers that we can see. So here is the unsimplified formula for this complex problem. (11 - a) + (11 - b) +(19 - c) - (11 - d) - (19 - e) = the answer when I add the three numbers on the bottom of the black dice and then subtract the sum of the numbers on the bottom of the two red dice. But watch how nicely it simplifies. In the end we have: 11 - (a + b + c) + (d + e) = your simple answer to a seemingly complex problem. Just start at 11 and subtract the sum of the three black dice and then add the sum of the two red dice. In fact, if you want to avoid negative numbers, just add the two red dice total to 11 before you subtract the three black dice total from it. Of course, it is possible that your answer could be negative, but players should be aware that numbers below zero exist and have to be dealt with before they are doing this sophisticated algebra. All while playing BottomsUp!
* Finally, BottomsUp with the green dice. Most players will realize it is easier to just figure out the numbers on the bottom and then multiply them than it is to do the algebra, but doing the algebra is great practice in multiplying binomials and will demonstrate that the famous FOIL (first, outer, inner, last) rule actually works. Here we go with the green (multiplying dice.) (11 - a) x (11 - b) = the product of the two numbers on the bottom. When we use our FOIL rule we find that 121 -11a -11b +ab = the product of the two numbers on the bottom. (Remember, that a and b are the variables for the top numbers, the numbers that we can see on the dice). So, (we are almost done) let’s say for example that our top numbers were a 5 and a 2. We can figure out that the number on the bottom of the 5 is 6 (11 minus 5 is 6) and the number on the bottom of the 2 is 9 (11 minus 2 is 9) and when we multiply 6 times 9 we get 54. Way easier than doing the algebra, but let’s do it anyway, just to demonstrate that this FOIL rule does work, and that there will be real life scenarios where using this algebraic formula will help. So here we go. 121 - (11 x 5) - (11 x 2) + (5 x 2) better equal 54. So, 121 - 55 - 22 + 10 better equal 54. Yahoo, it does! Better yet, the little BottomsUp game that you learned before you even knew that algebra existed or what a variable was has now demonstrated for you why “we even need to learn algebra.”

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